MATH 17 – QUIZ 4
4 OCTOBER 2010

Name: SOLUTION

NO CALCULATORS

1. Determine whether the following integral converges or diverges.

$$\int_1^\infty \frac{\sin x + 2}{x^2}\,dx$$

Since $-1 \le \sin x \le 1$, $1 \le \sin x + 2 \le 3$

So $\frac{\sin x + 2}{x^2} \le \frac{3}{x^2}$ for all x.

$\int_1^\infty \frac{3}{x^2}dx = \lim_{t\to\infty} \int_1^t \frac{3}{x^2}dx = \lim_{t\to\infty} -3x^{-1}\Big|_1^t = \lim_{t\to\infty} 3 - \frac{3}{t} = 3.$

Since $\int_1^\infty \frac{3}{x^2}\,dx$ converges, $\int_1^\infty \frac{\sin x+2}{x^2}\,dx$ $\boxed{\text{converges}}$ by the comparison test.

2. Solve the following differential equation, with initial condition $y(0) = 0$.

$$\sqrt{xy}\,\frac{dy}{dx} = \sin(\sqrt{x})$$

$\sqrt{y}\,\frac{dy}{dx} = \frac{\sin\sqrt{x}}{\sqrt{x}}$ so $\sqrt{y}\,dy = \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

$\int \sqrt{y}\,dy = \int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ $u = \sqrt{x}$, $du = \frac{1}{2}x^{-1/2}dx$

$\frac{2}{3}y^{3/2} = \int 2\sin u\,du = -2\cos\sqrt{x} + C.$

$y^{3/2} = -3\cos\sqrt{x} + C.$

$0^{3/2} = y(0)^{3/2} = -3\cos\sqrt{0} + C = -3 + C \longrightarrow C = 3.$

So $\boxed{y^{3/2} = -3\cos\sqrt{x} + 3}$